A Simple Online Problem

… I want to surveil the suspect …

… while saving the fundings ..

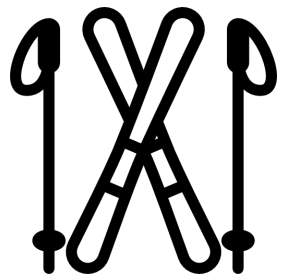
… will ski for an unknown number of days ..

A Simple Online Problem

… while saving the fundings ..

… will ski for n days ..

unknown

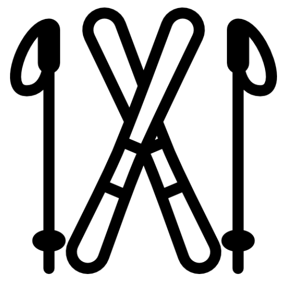


sale price: $100

rental price: $10

How to Make Decisions (Buy or Rent) Online?

The Ski-Rental Problem



sale price: $100

rental price: $10

… will ski for n days ..

unknown

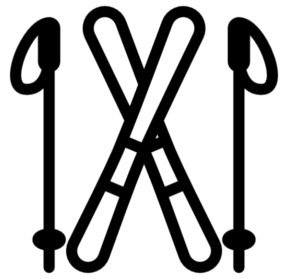
If n is known:

If $n < 10$, keep renting	Optimal Cost is $\$10n$
If $n \geq 10$, just buy a pair of skis	Optimal Cost is $\$100$

n is unknown: What should I do?

2-Competitive Algorithm

The Ski-Rental Problem



sale price: \$100

rental price: \$10

… will ski for n days ..

unknown

If n is known:

If $n < 10$, keep renting	Optimal Cost is $\$10n$
If $n \geq 10$, just buy a pair of skis	Optimal Cost is $\$100$

n is unknown:

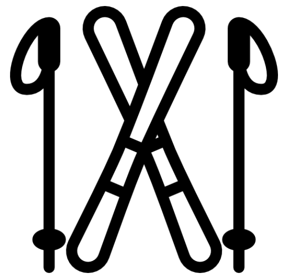
Buy on day 10 regardless

If $n < 10$,	Algorithm Cost is $\$10n$
If $n \geq 10$,	Algorithm Cost is $\$200$

$$\max_{n \geq 1} \frac{\text{Algorithm Cost}(n)}{\text{Optimal Cost}(n)} \leq 2$$

Competitive Ratio

The Ski-Rental Problem



sale price: \$100

rental price: \$10

… will ski for n days ..

↑ *unknown*

If n is known:

- If $n < 10$, keep renting — Optimal Cost is $\$10n$
- If $n \geq 10$, just buy a pair of skis — Optimal Cost is $\$100$

n is unknown:

Buy on day 10 regardless

- If $n < 10$, Algorithm Cost is $\$10n$
- If $n \geq 10$, Algorithm Cost is $\$200$

$$\text{Competitive Ratio} := \max_{n \geq 1} \frac{\text{Algorithm Cost}(n)}{\text{Optimal Cost}(n)} \leq 2$$

denoted by **CR**

From Classic to Learning-Augmented World

The Ski-Rental Problem

n is unknown:

Buy on day 10 regardless

If $n < 10,$ algorithm cost is $\$10n$

If $n \geq 10,$ algorithm cost is $\$200$

$$\text{Competitive Ratio} := \max_{n \geq 1} \frac{\text{Algorithm Cost}(n)}{\text{Optimal Cost}(n)} \leq 2$$

— This is a **simple** and **classic** online problem in computer science

— What if we get a machine learning prediction $\tilde{n}$ of n?

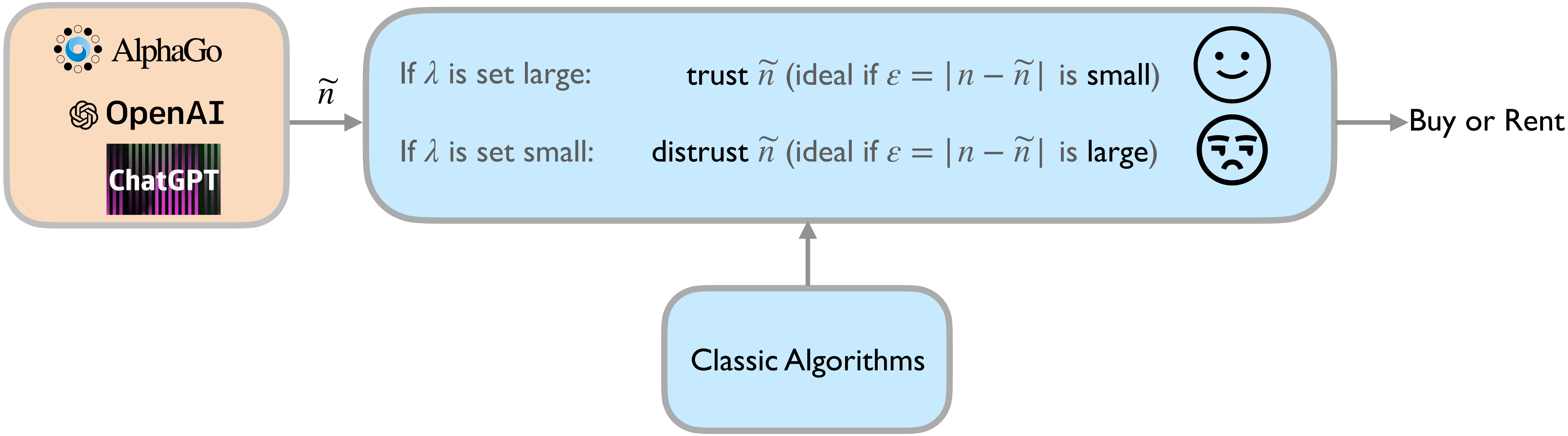
$n \quad \hat{n}$

- $\tilde{n}$ may not be trustworthy
- need to design new learning-augmented online algorithms

Combine Classic Algorithms with Machine Learning Outputs

Learning-Augmented Ski-Rental *[Kumar et. al. NeurIPS 2018]*

Idea: Introduce a **trust parameter** $\lambda \in (0,1)$



Combine Classic Algorithms with Machine Learning Outputs

Learning-Augmented Ski-Rental *[Kumar et. al. NeurIPS 2018]*

Idea: Introduce a trust parameter $\lambda \in (0,1)$

AlphaGo
OpenAI
ChatGPT

$\tilde{n}$ →

If $\tilde{n} \geq 100$ buy on day $\lceil 100(1-\lambda) \rceil$

If $\tilde{n} < 100$ buy on day $\lceil 100/(1-\lambda) \rceil$

→ Buy or Rent

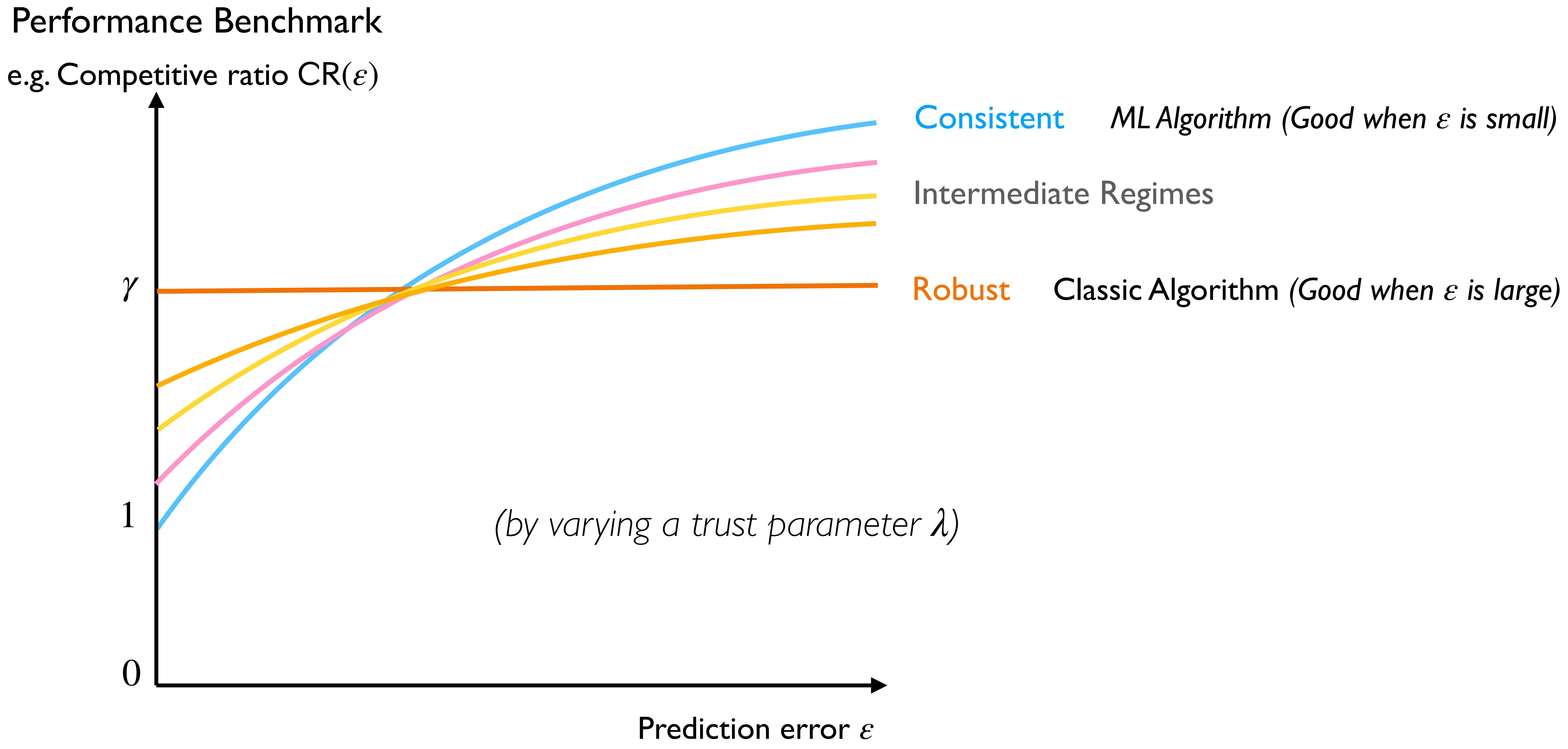
- Design a *meta-algorithm* based on the classic one with a tuning parameter λ
- Varying λ will achieve different results (Robustness and Consistency Trade-off)
- Competitive ratio depends on λ

$$\textbf{CR} \leq \min\left\{\frac{1+\lambda}{\lambda}, (1+\lambda) + \frac{\varepsilon}{(1-\lambda)\text{OPT}}\right\} \qquad \text{OPT} \equiv \min\{100, 10n\}$$

- Both deterministic (above) and stochastic versions
- The bounds are tight *[Wei et. al. NeurIPS 2020]*

The Goal of Learning-Augmented Algorithms

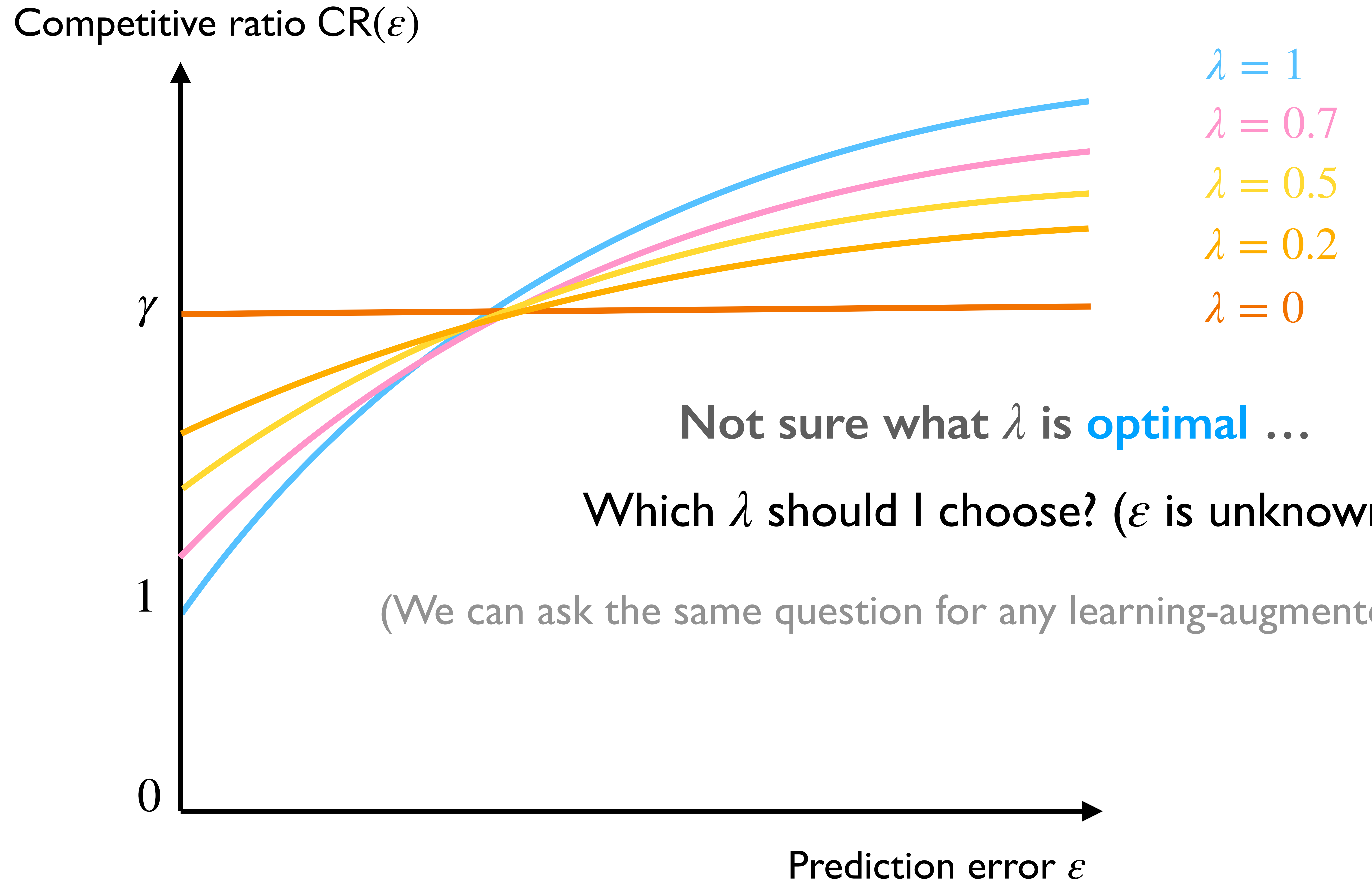
Meta-algorithms Consistency vs Robustness Trade-off



First Limitation

General Goal of Learning-Augmented Algorithms

Consistency vs Robustness Trade-off



Not sure what λ is optimal …

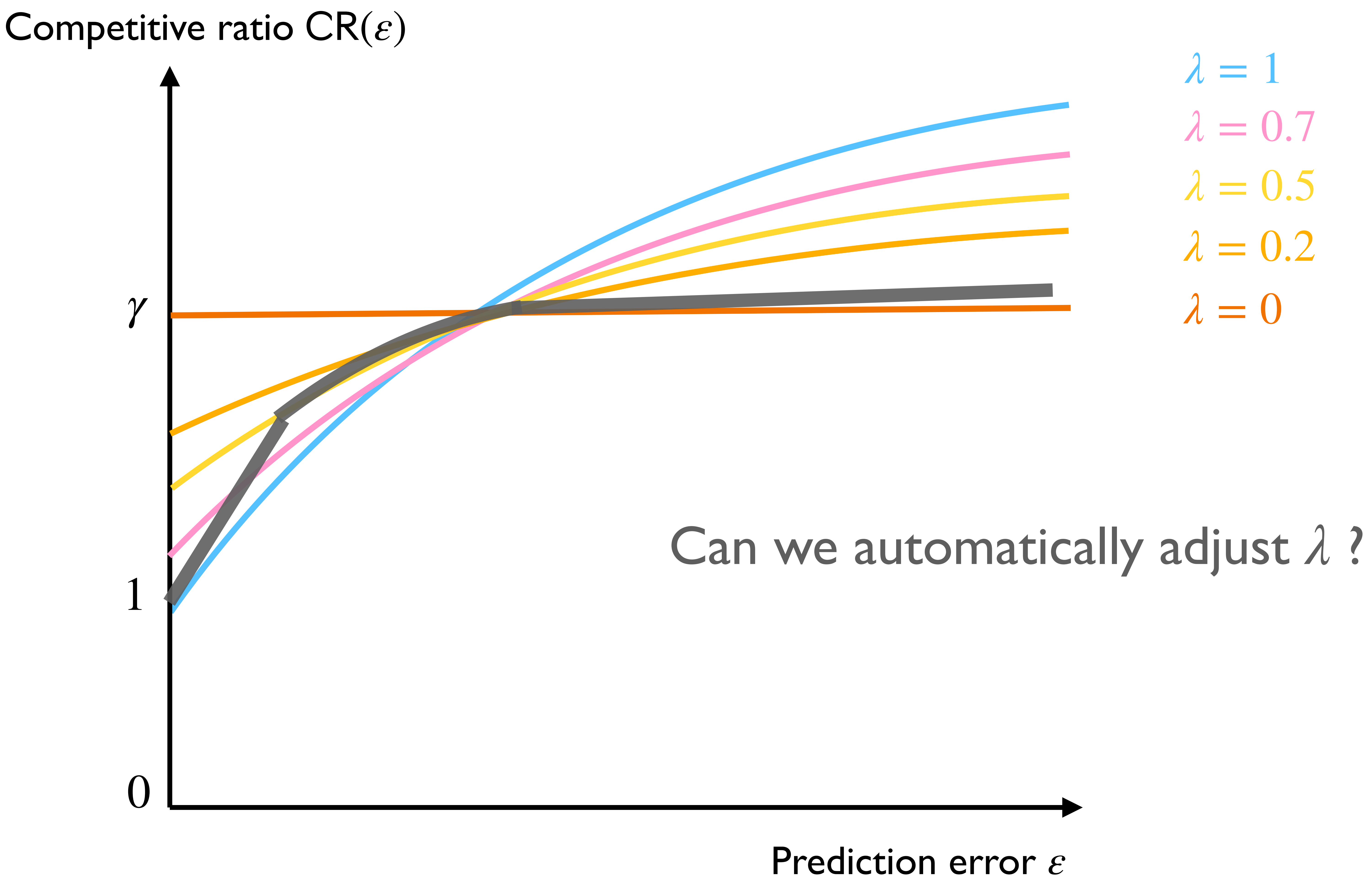
Which λ should I choose? (ε is unknown)

(We can ask the same question for any learning-augmented online algorithms)

First Limitation

Issue: Prediction error ε is not known a priori

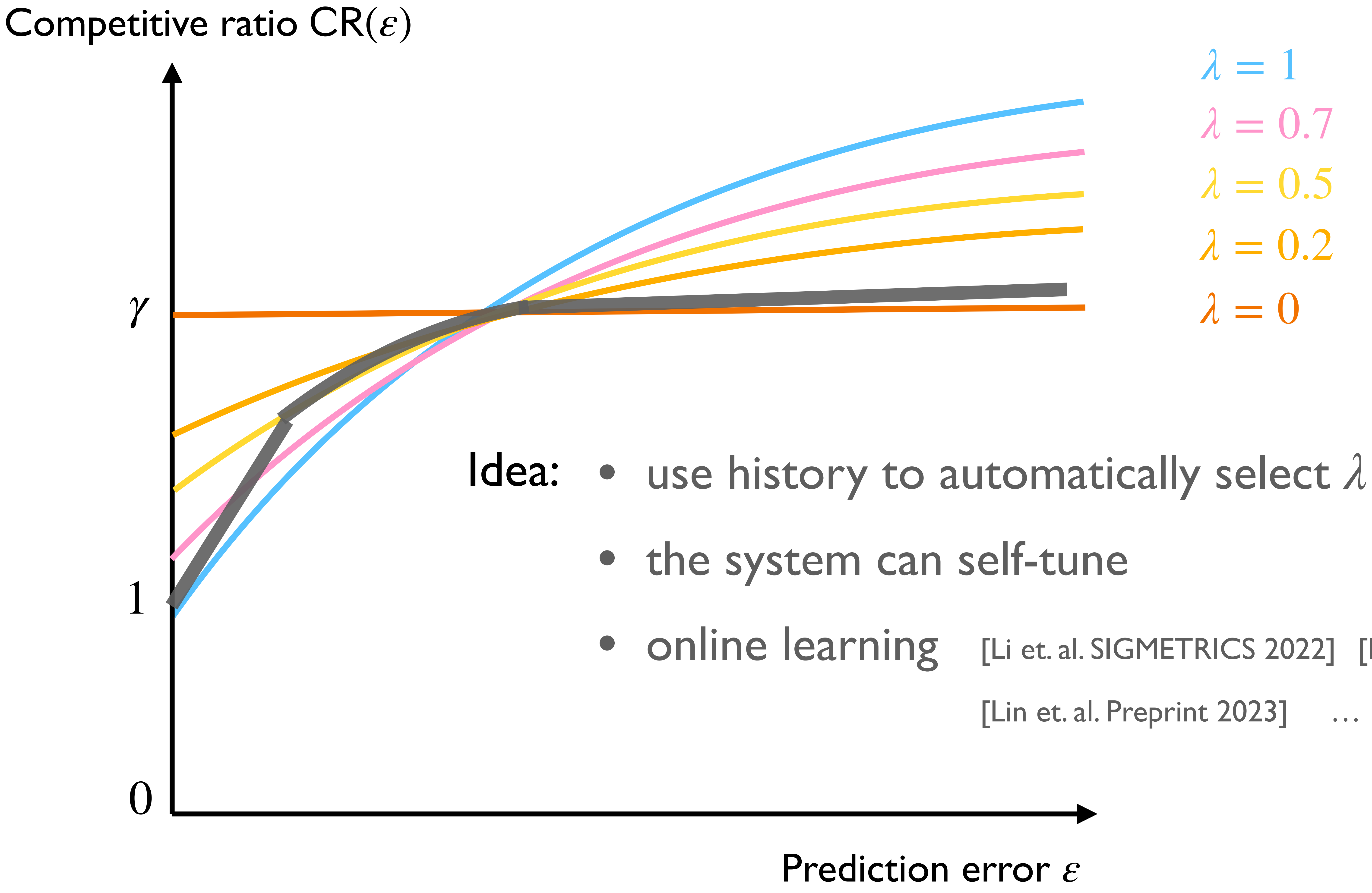
Goal: Find an online algorithm with good Competitive Ratio **CR** regardless of prediction error ε



One Solution: **Online Learning**

General Goal of Learning-Augmented Algorithms

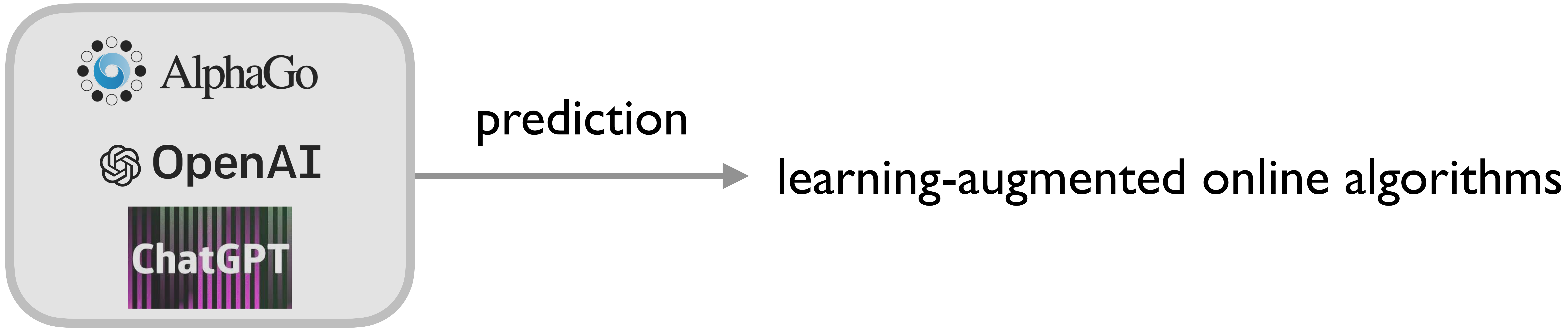
Consistency vs Robustness Trade-off



Idea:

- use history to automatically select λ
- the system can self-tune
- online learning [Li et. al. SIGMETRICS 2022] [Khodak et. al. NeurIPS 2022] [Lin et. al. Preprint 2023] … [Li et. al. NeurIPS 2024]

Second Limitation



- The machine learning tools are considered as **black-boxes**

- **Structural information** of the model and ML tools can be helpful
  - specific forms of predictions [Li et. al. SIGMETRICS 2022]
  - grey-box ML models (Q-value functions of value-based policies)

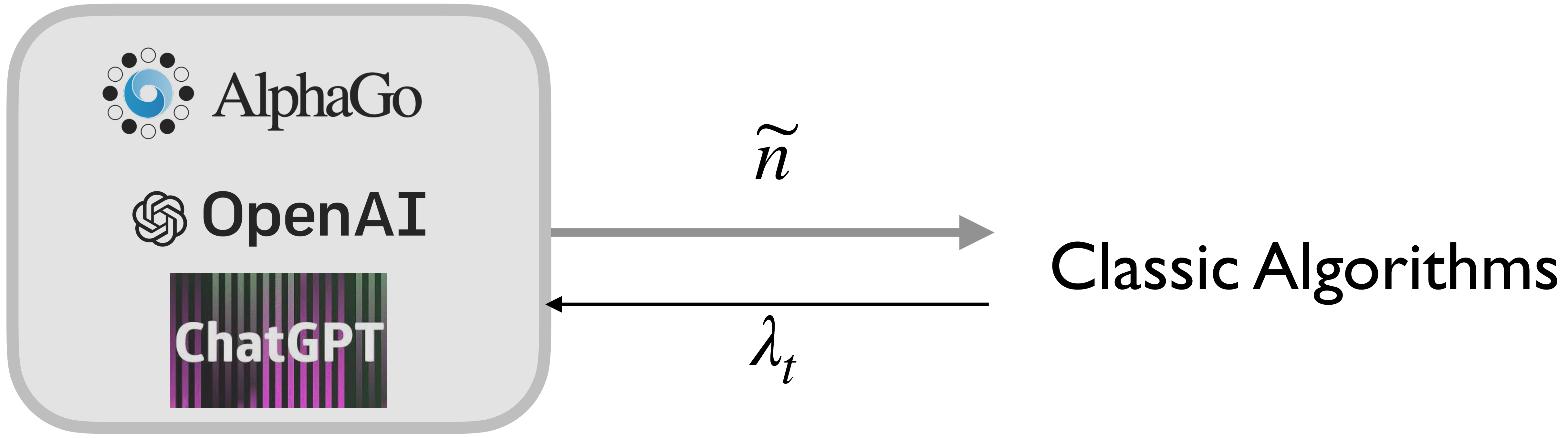
    [Li et. al. Preprint 2023]

  - can be used to self-tune λ (second solution)

Second Limitation

- Learning-augmented —> Learning-infused

- Q-learning
- Linear Regression
- Multi-arm bandit



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    [Li et. al. Preprint 2023]
  - can be used to self-tune λ (second solution)

Learning-Augmented Algorithms

Online Problems	**Imperfect Predictions**	
Ski-rental	Number of Skiing Days	[Wei et. al. NeurIPS 2020] [Purohit et. al. NeurIPS 2018]
Secretary Problem	Maximum Price	[Antoniadis et. al. NeurIPS 2020]
Online Bipartite Matching	Adjacent Edge-weights	
	Black-box AI/ML Advice	
Convex Body Chasing	Suggested Actions	[Christianson et. al. COLT 2022]
Online Subset Sum	Decision	[Xu et. al. Journal of Global Optimization 2022]
Online Set Cover	Predicted Covering	[Bamas et. al. NeurIPS 2020]
…	…	

Over 100 topics on this website: https://algorithms-with-predictions.github.io/

Learning-Augmented Algorithms

Online Problems	不准确预测 Imperfect Predictions	
Ski-rental	Number of Skiing Days	[Wei et. al. NeurIPS 2020] [Purohit et. al. NeurIPS 2018]
Secretary Problem	Maximum Price	[Antoniadis et. al. NeurIPS 2020]
Online Bipartite Matching	Adjacent Edge-weights	
Linear Quadratic Control	**System Perturbations**	**[Li et. al. SIGMETRICS 2022]** **[Li et. al. NeruIPS 2024]**
	不可信AI建议 Black-box AI/ML Advice	
Convex Body Chasing	Suggested Actions	[Christianson et. al. COLT 2022]
Online Subset Sum	Decision	[Xu et. al. Journal of Global Optimization 2022]
Online Set Cover	Predicted Covering	[Bamas et. al. NeurIPS 2020]
Q Learning	Q-Value Functions	[Golowich et. al. NeurIPS 2022]
Value-Based RL	**Q-Value Functions (灰盒)/Actions (黑盒)**	**[Li et. al. NeurIPS 2023]**
Stochastic Game	**Type Beliefs**	**[Li et. al. NeurIPS 2024]**
…	…	

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